Cross-focusing of two laser beams in a plasma

R. Bharuthram, and J. Parashar
Department of Physics, University of Durban–Westville, Private Bag X 54001, Durban 4000, South Africa
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Cross-focusing of two copropagating laser beams in a plasma is investigated using paraxial ray theory. If the lasers have a frequency difference equal to the electron plasma frequency, they can drive a large amplitude plasma wave. The ponderomotive force due to the plasma wave forces the plasma electrons outwards thereby generating a parabolic density profile giving rise to cross-focusing. The results show a decrease in threshold for focusing by two orders of magnitude as compared to focusing due to the ponderomotive force of the laser beams. [S1063-651X(99)01109-5]

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I. INTRODUCTION

Plasmas are rich in nonlinear phenomena, viz, parametric instabilities [1–3], harmonic generation [4–6], mode conversion [3,7,8], soliton formation [9,10], etc. There is much recent interest in nonlinear optics of plasmas, including wave mixing and plasma wave excitation [11–13]. One of the ways to excite a plasma wave is by beating two laser beams with their frequency difference equal to the electron plasma frequency [14]. It has been demonstrated experimentally that such electron plasma waves can be employed to accelerate charged particles up to 3.7 MeV of energies, and the device is known as a beat wave accelerator [15]. Under the resonance condition the electron plasma wave can attain a large amplitude and consequently its associated ponderomotive force on the electrons can be much larger compared to the ponderomotive force due to the laser beams. Earlier studies on focusing of laser beams have considered the effect of the ponderomotive force due to the laser beams [16,17]. In this paper we examine the cross-focusing of two laser beams under the resonance condition when the ponderomotive force due to an electron plasma wave is large compared to the ponderomotive force due to laser beams. The two lasers of frequencies ω_1 and ω_2 ($\omega_1-\omega_2=\omega_p$) and wave vectors $\vec{k}_1$ and $\vec{k}_2$ ($\vec{k}_1-\vec{k}_2=\vec{k}$) beat with each other and excite a large amplitude electron plasma wave at $(\omega,\vec{k})$. The electron plasma wave exerts a ponderomotive force on the electrons and forces them out, giving rise to a parabolic density profile responsible for cross-focusing.

The paper is organized as follows. In Sec. II we derive our nonlinear permittivity. We then solve the wave equation for electromagnetic waves under the WKB approximation following the technique used by Akhmanov, Sukhorukov, and Khokholov [18], and study the cross-focusing of the laser beams in Sec. III. Finally our results are presented and discussed in Sec. IV.

II. NONLINEAR PERMITTIVITY

Consider the propagation of two Gaussian laser beams through a collisionless plasma of density n_0^0, temperature T_c,

$$\vec{E}_j=\hat{y}A_j\exp[-i(\omega_j t-k_j z)],$$
$$A_j^2|_{z=0}=A_{j0}^2\exp(-r^2/r_0^2),\quad j=1,2 \tag{1}$$

$k_j\approx(\omega_j/c)(1-\omega_{p0}^2/\omega_j^2)^{1/2}$ is the propagation constant in the linear approximation. r_0 the initial beam radius, $\omega_{p0}^2=(4\pi n_0^0 e^2/m)^{1/2}$ is the unmodified plasma frequency, and e and m are the electronic charge and mass. We shall take $\omega_1-\omega_2=\omega_{p0}\ll\omega_1,\omega_2$. For $z>0$, we assume energy conserving Gaussian intensity profiles, with spot sizes r_0f_j and axial amplitudes A_{j0}/f_j,

$$|A_j|^2=\frac{A_{j\,0}^2}{f_j^2}\exp(-r^2/r_0^2f_j^2), \tag{2}$$

where f_j is known as the beam width parameter. The lasers impart oscillatory velocities to the electrons given by

$$\vec{v}_j=\frac{e\vec{E}_j}{mi\omega_j}. \tag{3}$$

$\vec{v}_1$ and $\vec{v}_2$ couple with the magnetic field of the lasers to exert a ponderomotive force $\vec{F}_{p\omega}$ on the electrons at ($\omega=\omega_1-\omega_2$, $\vec{k}=\vec{k}_1-\vec{k}_2$):

$$\vec{F}_{p\omega}=e\vec{\nabla}\phi_{p\omega}=-\frac{m}{2}\vec{\nabla}\vec{v}_1\cdot\vec{v}_2^*, \tag{4}$$

where $*$ denotes the complex conjugate.

The electron velocity due to $\phi_{p\omega}$ and the self-consistent potential ϕ are found to be

$$\vec{v}\,'=-\frac{e}{mi\omega}\vec{\nabla}(\phi+\phi_{p\omega}). \tag{5}$$

Using Eq. (5) in the continuity equation, $\partial n/\partial t_l+\vec{\nabla}\cdot(n\vec{v})=0$, we obtain the electron density perturbation,

$$n_1=\frac{n_0}{\omega}\vec{k}\cdot\vec{v}\,', \tag{6}$$

where n_0 is the static component of electron density. It is different from n_0^0 due to the modification caused by static ponderomotive force.

Using Eq. (6) in the Poisson equation $\nabla^2\phi=4\pi n_1 e$ we get

$$\epsilon\phi=-\chi_c\phi_{p\omega}, \tag{7}$$

where $\epsilon=1+\chi_e$, $\chi_e=-\omega_p^2/\omega^2$.

Using Eqs. (4) and (7) in Eq. (5) we obtain the electron velocity at $(\omega,\vec{k})$,

$$\vec{v}=-\frac{e^2k}{2m^2\omega\epsilon}\frac{\vec{E}_1\cdot\vec{E}_2^*}{\omega_1\omega_2}\hat{z}. \tag{8}$$

$\vec{v}_1$, $\vec{v}_2$, and $\vec{v}$ exert a dc ponderomotive force $\vec{F}_{p0}=e\vec{\nabla}\phi_{p0}$ on the electrons:

$$\phi_{p\,0}=-\frac{m}{2e}(\vec{v}_1\cdot\vec{v}_1^*+\vec{v}_2\cdot\vec{v}_2^*+\vec{v}\cdot\vec{v}^*). \tag{9}$$

As the electrons move radially outward due to the ponderomotive force, they produce a space charge field $\vec{E}_s=-\vec{\nabla}\phi_s$, that pulls the ions along. In the steady state the equations for electron and ion motions can be written as

$$\vec{0}=-e\vec{\nabla}(\phi_s+\phi_{p0})-\frac{T_e}{n_e}\vec{\nabla}n_e$$

and

$$\vec{0}=-e\vec{\nabla}\phi_s-\frac{T_i}{n_i}\vec{\nabla}n_i, \tag{10}$$

respectively. Here n_e and n_i are modified static densities of electrons and ions. Adding these equations and employing the quasineutrality condition $n_e=n_1=n_0$ we obtain

$$\Delta n=n_0-n_0^0=-\frac{n_0^0e\phi_{p0}}{T_e+T_i}. \tag{11}$$

The plasma permittivity at ω, $\vec{k}$ can be written as

$$\epsilon=1-\frac{\omega_{p\,0}^2}{\omega^2}+\frac{\omega_{p0}^2}{\omega^2}\alpha, \tag{12}$$

where

$$\alpha=-\frac{\Delta n}{n_0^0}=\frac{e^2}{2(T_e+T_i)m}\left[-\frac{A_{10}^2e^{-r^2/r_0^2f_1^2}}{\omega_1^2f_1^2}+\frac{A_{20}^2e^{-r^2/r_0^2f_2^2}}{\omega_2^2f_2^2}+\frac{k^2e^2A_{10}^2A_{20}^2}{4\omega^2\epsilon^2m^2\omega_1^2\omega_2^2}\frac{e^{-r^2/r_0^2(1/f_1^2+1/f_2^2)}}{f_1^2f_2^2}\right]. \tag{13}$$

Under the condition $\omega_1-\omega_2=\omega_{p0}$, $\epsilon\to 0$, consequently the first two terms in Eq. (13) can be neglected as compared to the third term, giving

$$\alpha\approx\frac{v_{10}^2v_{20}^2}{4(1+T_i/T_e)v_{\rm th}^2c^2}\frac{1}{f_1^2f_2^2\epsilon^2}e^{-(r^2/r_0^2)(1/f_1^2+1/f_2^2)}, \tag{14}$$

where $v_{0j}^2=e^2A_{j0}^2/m^2\omega_j^2$, $j=1,2$ and $v_{\rm th}^2=2T_e/m$ and $\omega/k\approx c$ has been used. v_{01},v_{02} are the magnitudes of electron oscillatory velocities due to the two laser beams at $r=0$, $z=0$. The plasma permittivity $\epsilon(\omega)$ can be written as [cf. Eq. (12)]

$$\epsilon\approx\frac{\omega_{p0}^2}{\omega^2}\alpha. \tag{15}$$

Using Eqs. (14) and (15) we get

$$\alpha=\left(\frac{v_{01}^2v_{02}^2}{4v_{\rm th}^2c^2(1+T_i/T_e)f_1^2f_2^2}\right)^{1/3}\left[1-\frac{1}{3}\frac{r^2}{r_0^2}\left(\frac{1}{f_1^2}+\frac{1}{f_2^2}\right)\right]. \tag{16}$$

The effective plasma permittivity at ω_1 can now be written as

$$\epsilon_1=1-\frac{\omega_{p0}^2}{\omega_1^2}-\frac{\omega_{p0}^2}{\omega_1^2}\alpha. \tag{17}$$

Introducing $I_1=cA_{0l}^2/8\pi$ and $I_2=cA_{02}^2/8\pi$ as the axial laser intensities, one may rewrite ϵ_1 as

$$\epsilon_1=1-\frac{\omega_{p0}^2}{\omega_1^2}-\frac{\omega_{p0}^2}{\omega_1^2}\left(\frac{4I_1I_2\pi^2e^4}{f_1^2f_2^2m^2\omega_1^2\omega_2^2(T_e+T_i)c^4}\right)^{1/3}\times\left[1-\frac{r^2}{3r_0^2}\left(\frac{1}{f_1^2}+\frac{1}{f_2^2}\right)\right]. \tag{18}$$

For $\omega_p=\omega_1-\omega_2\ll\omega_1,\omega_2$, we have $\omega_1\sim\omega_2$ and $f_1=f_2$. Under this approximation Eq. (18) can be rewritten as

$$\epsilon_1=\epsilon_{10}-\epsilon_{11}r^2, \tag{19}$$

where

$$\epsilon_{10}=1-\frac{\omega_{p0}^2}{\omega_1^2}-\frac{\omega_{p0}^2}{\omega_1^2}\left(\frac{4I_1I_2\pi^2e^4}{m^2\omega_1^2(T_e+T_i)c^4}\right)^{1/3}\frac{1}{f_1^{4/3}},$$

$$\epsilon_{11}=\left(\frac{4I_1I_2\pi^2e^4}{m^2\omega_1^2(T_e+T_i)c^4}\right)^{1/3}\frac{2}{3r_0^2}\frac{1}{f_1^{10/3}}.$$

In the case of a single laser beam, the effective plasma permittivity (for ponderomotive nonlinearity) can be written from Eq. (13) by taking $A_{20}\to 0$ as

$$\epsilon_1=1-\frac{\omega_{p0}^2}{\omega_1^2}+\frac{\omega_{p0}^2}{\omega_1^2}\frac{8\pi e^2 1_1(1-r^2/r_0^2f_1^2)}{2c\omega_1^2f_1^2m(T_e+T_i)}.$$

III. CROSS-FOCUSING

The wave equation governing the propagation of an electromagnetic wave is

$$\nabla^2\vec{E}_1+\frac{\omega_1^2}{c^2}\epsilon_1\vec{E}_1=0. \tag{20}$$

Writing the t and fast z dependence of $\vec{E}_1$ as given by Eq. (1), using Eq. (12), and assuming $|\vec{\nabla}\epsilon/\epsilon| \ll k$, Eq. (13) can be written, in the WKB approximation, as

$$2ik_1\frac{\partial A_1}{\partial z}+\nabla_\perp^2 A_1-\frac{\omega_{p0}^2}{c^2}(\epsilon_{10}-\epsilon_{11}r^2)A_1=0, \tag{21}$$

where

$$\nabla_\perp^2 \equiv \frac{l}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial r^2}.$$

We introduce an eikonal $A_1=A_{10}e^{ikS}$ and separate the real and imaginary parts of Eq. (14), giving

$$\frac{\partial A_{10}^2}{\partial z}+\frac{\partial S}{\partial r}\frac{\partial A_{10}^2}{\partial r}+A_{10}^2\nabla_\perp^2 S=0 \tag{22}$$

and

$$2k_1\frac{\partial S}{\partial z}-\left(\frac{\partial S}{\partial r}\right)^2=\frac{1}{A_{10}}\nabla_\perp^2 A_{10}+\frac{\omega_{p0}^2}{c^2}(\epsilon_{10}-\epsilon_{11})r^2. \tag{23}$$

Assuming a Gaussian ansatz for A_{10}^2 [cf. Eq. (2)], and substituting in Eqs. (22) and (23) we get

$$2k_1\frac{\partial S}{\partial z}+\left(\frac{\partial S}{\partial r}\right)^2=-\frac{1}{r_0^2 f_1^2}\left(2-\frac{r^2}{r_0^2 f_1^2}\right)+\frac{\omega_{p0}^2}{c^2}(\epsilon_{10}-\epsilon_{11}r^2) \tag{24}$$

and

$$\nabla_\perp^2 S-\frac{\partial S}{\partial r}\frac{2r}{r_0^2 f_1^2}-\frac{2k_1}{f_1}\left(1-\frac{r^2}{r_0^2 f_1^2}\right)\frac{\partial f_1}{\partial z}=0, \tag{25}$$

respectively.

We expand S in the paraaxial ray approximation as

$$S=\Psi(z)+\beta\frac{r^2}{2}. \tag{26}$$

Using Eq. (26) in Eqs. (24) and (25), collecting various powers of r, and solving we obtain the equation governing the beam width parameter:

$$\frac{d^2 f_1}{d\xi^2}=\frac{1}{f_1^3}-\frac{2}{3}\frac{\omega_{p0}^2}{c^2k_1^2}\frac{R_d^2}{r_0^2}\left(\frac{v_{01}^2 v_{02}^2}{4v_{\mathrm{th}}^2 c^2}\right)^{1/3}\frac{1}{f_1^{7/3}}, \tag{27}$$

where $R_d=k_1 r_0^2$, $\xi=z/R_d$.

The first term on the right hand side corresponds to the diffraction divergence of the beam while the second term corresponds to the nonlinear refraction. The threshold power for focusing may be obtained by demanding $df_1/d\xi=0$, $f_1=1$ for all values of ξ [16]. For powers above threshold, $d^2 f_1/d\xi<0$ and f_1 decreases as the beam propagates and attains a minimum value $f_{1\,\min}$ at a certain value of ξ, after which f_1 starts increasing again as diffraction effects become more pronounced than the nonlinear refraction effect. We have solved Eq. (27) numerically and displayed the variation of f_1 with ξ in Fig. 1.

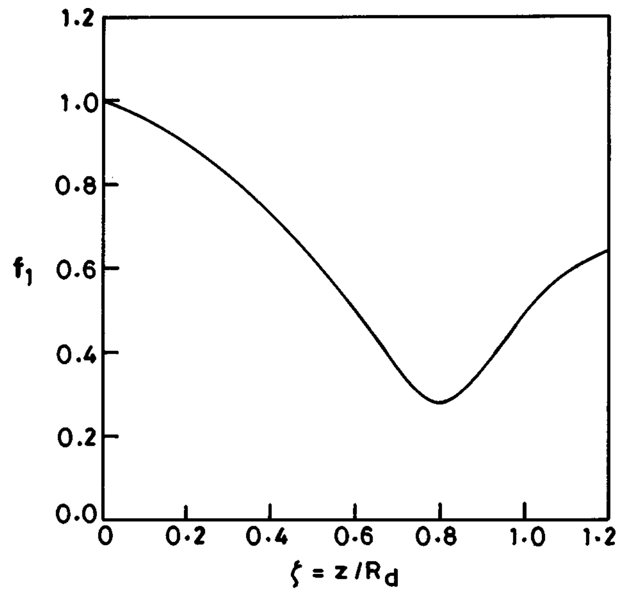

FIG. 1. Variation of beam width parameter with dimensionless distance of propagation $\xi(=z/R_d)$.

IV. RESULTS AND DISCUSSION

Electron plasma waves can be resonantly excited by beating of two copropagating laser beams. The plasma wave exerts a ponderomotive force on the electrons thereby creating a density depression which focuses the laser beams. For the following parameters: plasma density $n\sim 4\times 10^{16}\,\mathrm{cm}^{-3}$, electron temperature of 100 eV, lasers of power density $\sim 10^{12}\,\mathrm{W/cm}^2$, spot size $r_0\sim 12\,\mu$, $\omega_1=10^{15}$ rad/sec, and $\omega_2=0.9\times 10^{15}$ rad/sec we find that beam width parameter decreases with the distance of propagation to a value of $f_{1\,\min}=0.28$ at $z=0.8R_d$ as shown in Fig. 1. Beyond this value of z the diffraction effects dominate over the self-focusing effect and the beam starts diverging. From Eq. (27) one may note that if laser beam 2 is weak and beam 1 is strong then both the beams can be focused by the plasma wave produced by beating them, if $\omega_1-\omega_2=\omega_p$. Calculations show that the threshold power for the focusing is reduced by a factor of 10^2 as compared to that obtained by considering the ponderomotive force due to the laser beams only (Liu and Tripathi [16] and Sodha, Ghatak, and Tripathi [17]). The axial power flow density of the plasma wave $I_p=\omega(\partial\epsilon/\partial\omega)(k^2\phi^2/8\pi)v_g$ where $v_g\simeq c^2/v_{\mathrm{th}}$ is the group velocity of the Langmuir wave. On using Eqs. (4) and (7) one obtains

$$\frac{I_p}{I_1}=\left(\frac{v_1}{c}\right)^{2/3}\left(\frac{v_{\mathrm{th}}}{c}\right)^{8/3}\frac{\omega_p^2}{4\omega_1^2},$$

which is quite small. Hence the depletion of the lasers due to the excitation of the plasma wave is negligible.

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